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NOTE ON STELLAR PERTURBATIONS OF NEARLY PARABOLIC ORBITS.

By E. Öpik.

NOTE ON STELLAR PERTURBATIONS OF NEARLY PARABOLIC ORBITS.

By E. Öpik.

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The following considerations refer to the question of statistical stability of cometary or meteoric orbits in the solar system, and partly also to the stability of wide double stars as far as perturbations by passing stars are concerned. The probability of ejection of a member out of the solar system, and the probability of getting out of the region of observability of the terrestrial observer will be discussed. Certain simplifications have been introduced, and on this account the estimates are approximate, being supposed to give the order of magnitude only.

An object (which we will call the meteor for the sake of simplicity) of very elongated orbit will spend most of its time near aphelion, and the stellar perturbations are there the strongest and the most frequent. For the order of magnitude it is therefore legitimate to consider only perturbations caused when the meteor is in aphelion. Assuming for the calculation of the frequency of perturbations that the meteor remains *all* the time in aphelion, we slightly overestimate the effect of perturbations, without much affecting the order of magnitude; on the other hand, the probable density of stars in space may be underestimated, because the number of white dwarfs, which may be relatively massive, is not known and is not taken into account here. This underestimate may balance the overestimate mentioned above.

The following considerations are valid when the effective duration of approach of star and meteor, defined later on, is small compared with the period of revolution of the meteor. This condition is easily fulfilled in all cases of practical interest, because of the high average velocity of the stars compared with the aphelion velocity of the meteor. Neglecting very close approaches of stars to the sun, the path of the star, on account of its high velocity, may be assumed to be rectilinear, with respect to the sun as well as to the meteor; the latter, during the short time of passage of the star, may be considered as practically at rest.

By reason of symmetry, after the passage is over, all the gravitational impulses parallel to the path of the star will be cancelled, and the resultant impulse contributed by the star to the meteor will be directed perpendicularly to the rectilinear path, toward the point of closest approach. Let us denote by ξ the distance of closest approach of star to meteor, expressed in astronomical units; by v the velocity of the star relative to the sun, expressed in units of the orbital velocity of the earth; by α the angle between the line joining star and meteor, and the direction of closest approach. Choosing as unit of time $\frac{1}{2\pi}$ years, and assuming the mass of the star equal to the sun's mass (which may be near the average), the gravitational acceleration exerted by the star upon the meteor will be $\frac{\cos^2 \alpha}{\xi^2}$; the component of this force parallel to the line of closest approach is

$$F_a = \frac{\cos^3 \alpha}{\xi^2},$$

and the total impulse contributed to the meteor along this line is

$$\Delta u = 2 \int_0^\infty F_a dt,$$

the time t being reckoned from the moment of closest approach. On the other hand,

$$v = \frac{dx}{dt} \quad \text{where} \quad x = \xi tg\alpha,$$

whence

$$dt = \frac{\xi d\alpha}{v \cos^2 \alpha}.$$

This gives

$$\Delta u = \frac{2}{\xi v} \int_0^{\pi/2} \cos \alpha \, d\alpha$$

or

$$\Delta u = \frac{2}{\xi v}. \tag{1}$$

The geometrical sum of the original velocity of the meteor and of Δu gives the perturbed velocity; the latter determines all the elements of the changed orbit.

The effective duration of approach, t_0, may be defined as the time necessary to produce the whole effect, Δu, by the star placed at the point of closest approach; thus

$$\frac{1}{\xi^2} t_0 = \Delta u,$$

whence

$$t_0 = \frac{2\xi}{v}. \tag{2}$$

One of the conditions of validity of (1) is that the period of revolution of the meteor, P, must be great as compared with t_0. For a very elongated orbit we may put

$$P = 2\pi \cdot \left(\frac{R}{2}\right)^{3/2}$$

in the adopted units of time, whence

$$\frac{P}{t_0} = \frac{\pi v}{2\sqrt{2}} \frac{R^{3/2}}{\xi}. \tag{3}$$

This ratio must be great—at least ten or more. The other condition for the practical validity of our considerations is that the eccentricity be great, that is, greater than 0.9.

For the average velocity of a star relative to the sun, we assume $v = 1 = 30$ km/sec., which is approximately the observed average. In this special case our formulae are

$$\Delta u = \frac{2}{\xi} \tag{1'}$$

and

$$\frac{P}{t_0} = 1,11 \frac{R^{3/2}}{\xi}. \tag{3'}$$

For a highly eccentric orbit the aphelion velocity is very small compared with the velocity of escape at aphelion distance; when considering the probability of the ejection of the meteor out of the solar system, we may neglect altogether the aphelion velocity, assuming it equal to zero. The condition for ejection is, then, that the relative perturbation, or the geometrical difference of the perturbation vectors

Δu_m and Δu_s for meteor and sun, be equal or greater than the velocity of escape, given by

$$v_e = \sqrt{\frac{2}{R}} \tag{4}$$

in the units chosen here. The condition of ejection is

$$\Delta u \geq v_e, \tag{5}$$

Δu being the absolute value of the relative perturbation vector for meteor and sun. It is easy to show that when R is great, it is necessary for ejection that the inequality

$$\frac{\xi_s}{R} \quad \text{or} \quad \frac{\xi_m}{R} \leq \sqrt{\frac{2}{R}} \tag{6}$$

be fulfilled; this follows when for a given value of ξ_m we consider the case of strongest effect, when the star passes through the line joining m and s between them, and when $\xi_s = R - \xi_m$. Here it is assumed that $\xi_m < \xi_s$. When $\xi_m > \xi_s$, we obtain the same result by interchanging the places of meteor and sun. When $\xi_m > R$ ($\xi_s > R$ also), the absolute sum of the two vectors

$$|\Delta u_m| + |\Delta u_s| = \frac{2}{\xi_m} + \frac{2}{\xi_s} < \frac{4}{R},$$

is smaller than v_e for $R > 8$; thus ejection cannot take place. From (6) we infer that for R exceeding about 100 astronomical units the ratio $\frac{\xi_m}{R}$ is small, whence ξ_m is small compared with R, and thus small relative to ξ_s also, and in (5) we may neglect Δu_s altogether. Ejection may happen only when the passage is so close to one of the two bodies that the relative perturbation of the other may be neglected. This gives the following simple form for the condition of ejection:

$$\Delta u_m = \frac{2}{\xi_m} \geq \sqrt{\frac{2}{R}}, \quad \text{or} \quad \xi_m \leq \sqrt{2R};$$

the effective radius of approach for ejection is thus

$$\xi_e = \sqrt{2R}. \tag{7}$$

The validity ratio (3′) in this case is

$$\frac{P}{t_0} \geq 1,11 \frac{R^{3/2}}{\sqrt{2R}} = 0,78R,$$

which is large for all possible values of R that may be of practical interest in our problem.

Assuming an average of twenty-five stars of mass 1 per 1000 cubic parsecs, or 3.10^{-18} per cubic astronomical unit, and taking into account that the total target area for ejection is twice $\pi\xi_e^2$ (because a close approach may happen with respect to the sun as well as with respect to the meteor, and because these two target areas do not overlap when $\xi_e < \frac{1}{2}R$, or when $R > 8$), we find for the probability of ejection within Δt years the value

$$3.10^{-18} \cdot 2\pi\xi_e^2 \cdot 2\pi\Delta t; \tag{8}$$

the last factor, 2π, is the number of astronomical units traveled by the star in one year. After substituting (7), the differential expression (8) leads to the following finite expression for the probable fraction, p_s, of meteors not ejected, having an average aphelion distance R a. u., after an interval of time equal to t years:

$$p_s = e^{-2,\,4.10^{-16}Rt}. \tag{9}$$

This formula provides us with a means of estimating the maximum possible limit of the solar system. Substituting for t a value near 3.10^9, the probable age of the solar system, we find for the probable fraction of meteors retained since the origin

$$p_s = e^{-10^{-6}R}.$$

The effective limit, corresponding to a fraction $1/e$ retained out of the original number may be put thus at $R_e = 10^6$ a. u. $= 5$ parsecs. The effective volume of the solar system is about 600 cubic parsecs, or about fifteen times greater than the volume per average solar mass in space.

This result holds for the assumption of a constant density of stars in space; the average density of stars along the path traveled by the sun since the origin of the solar system might have been different from the present density; there may be a variation with time also. That is, if the galactic system has been expanding appreciably, the average density of stars in space must have been greater in the past

than it is now, and the effective radius, R_e, may be assumed to have been somewhat smaller. Taking into account this circumstance, we may estimate the possible limit of the solar system at from 2.10^5 to 10^6 astronomical units. The period of revolution for these extreme distances will then be equal to from 3.10^7 to 4.10^8 years; thus, even the most remote present members of the solar system must have had time to complete several revolutions since the time of their origin.

The quantitative conditions of ejection, derived here for nearly parabolic orbits, will not change very much, as far as the order of magnitude is concerned, for less eccentric orbits nor even for nearly circular ones. Thus, the effective limits set here may be regarded as of general validity.

For a star of mass μ times the solar mass, the exponent in formula (9) is to be divided by μ; for the same age, t, the extreme limit of the system R_e is to be multiplied by μ. In this form our considerations may be applied to the question of stability of wide double stars—the so-called pairs with common proper motion. For $\mu = 1$ we find the time of retaining the fraction $1/e$ of companions at an aphelion distance R as follows:

R, a. u.	10^6	10^5	10^4	10^3
t, years	3.10^9	3.10^{10}	3.10^{11}	3.10^{12}

For other values of the mass these intervals of time are to be multiplied by μ. It may be shown that the average velocity of ejection is such that within the time t, for $R < 10^6$, the separation of the two former members of the system becomes much greater than fifty parsecs or 10^7 a. u. so that in most cases an ejection results in practical separation; the former components will not be counted any more as members of the same system.

In the preceding it was assumed that the orbit, or at least the aphelion distance, remains substantially unchanged until a close approach throws the meteor out. This assumption is not exactly true. Less powerful, but more numerous perturbations will have the effect of increasing the major axis and the average aphelion distance, thus making the final ejection more easy. The rate of ejections is therefore somewhat underestimated above; the order of magnitude, however, is not affected. As to the effective limit of the solar system, the estimate need not be changed, because the shifting outward of R will increase the limit and counterbalance the effect of a greater number of ejections. The conclusion is that within intervals comparable with the

age of the solar system, meteors or comets remain practically well connected with the sun even at distances as great as the average separation between two stars in space. However, from the standpoint of observability their detection becomes impossible when the perihelion distance exceeds 1 for meteors, and is a small multiple of 1 for comets. The perturbations of the perihelion distance are thus the most important ones from the practical standpoint. The question of persistence of gaseous constituents in meteors, in connection with the appearance of comet tails and with the determination of the age of meteorites from the helium content, also depends chiefly upon the perihelion distance, which determines the maximum temperature attained by the meteor during its revolution around the sun.

The law of conservation of angular velocity gives for a nearly parabolic orbit, when the perihelion velocity is assumed equal to the velocity of escape, $\sqrt{\frac{2}{q}}$,

$$q = \frac{R^2 v_a^2}{2}; \tag{10}$$

here q is the perihelion distance, R the aphelion distance, v_a the aphelion velocity (units same as before). Thus q is very sensitive to changes in v_a, being proportional to the square of the latter. The calculation of the perturbational vector in the present case is more complicated than in the former, because we cannot neglect one of the two impulses, Δu_m and Δu_s. Without going into detail, we may state that the absolute value of the relative impulse meteor *minus* sun is given by

$$\Delta u = \frac{2R \cdot |\sin \gamma|}{\xi_m \sqrt{\xi_m^2 + R^2 \sin^2 \gamma + 2R\xi_m (\cos \alpha \cos A + \cos \beta \sin A)}}; \tag{11}$$

here α, β, γ are the angles formed by the vector R with the x, y, z axes of Cartesian coördinates; the z axis is assumed to coincide with the path of the perturbing star, and the meteor is placed in the xy plane; A is the azimuthal angle of the line of closest approach to meteor, recorded from the x axis in the xy plane.[1] The independent

[1] The derivation of (11) is simple in Cartesian coördinates, where

$$\Delta u_x = \frac{2x_m}{\xi_m^2} - \frac{2x_s}{\xi_s^2}; \quad \Delta u_y = \frac{2y_m}{\xi_m^2} - \frac{2y_s}{\xi_s^2}; \quad \Delta u_z = 0;$$

and

$$\Delta u = \sqrt{\Delta u_x^2 + \Delta u_y^2 + \Delta u_z^2}.$$

variables are ξ_m, R, and the direction of R defined by α, β, and γ under the restriction $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. From (11) it is possible to find the frequency of different values of Δu. The exact procedure, however, is too complicated; in this preliminary discussion we will substitute in (11), for the trigonometric functions, their average values calculated on the assumption of a random orientation of the vector R. In this way we obtain a certain approximate expression for the average value $\overline{\Delta u}$ of the vector Δu. For the function (11) this procedure means underestimating $\overline{\Delta u}$. Further, let Δv_a be the component of Δu along v_a, so that

$$\Delta v_a = \Delta u \cos\varphi, \tag{12}$$

where φ is the angle between Δu and v_a. Assuming a random frequency for this angle,[1] we have $|\overline{\cos\varphi}| = \frac{1}{2}$. Also, in (11), $|\overline{\sin\gamma}| = \frac{\pi}{4}$; $\overline{\sin^2\gamma} = \frac{2}{3}$; $\overline{\cos\alpha} = \overline{\cos\beta} = 0$. This gives for the order of magnitude of the average perturbation in v_a the expression

$$\overline{\Delta v_a} = \pm\frac{\pi}{4}\,\frac{R}{\xi_m\sqrt{\xi_m^2 + \frac{2}{3}R^2}}. \tag{13}$$

From (10) we find the expression for the changed perihelion distance

$$q_1 = \frac{R^2(v_a + \Delta v_a)^2}{2} = \frac{R^2 v_a^2}{2} + \frac{R^2}{2}(\Delta v_a)^2 + R^2 v_a \Delta v_a$$

and with the aid of (10) again

$$q_1 - q = \frac{R^2}{2}(\Delta v_a)^2 + R\sqrt{2q}\,\Delta v_a. \tag{14}$$

Substituting for Δv_a its average value from (13), retaining the $\pm$ sign, and putting $\varepsilon = \xi_m/R$, we get the average perturbation of q as follows:

$$\bar{q}_1 - q = \frac{\pi^2}{32}\cdot\frac{1}{\varepsilon^2(\varepsilon^2 + \frac{2}{3})} \pm \frac{\pi\sqrt{2q}}{4}\cdot\frac{1}{\varepsilon\sqrt{\varepsilon^2 + \frac{2}{3}}}. \tag{15}$$

The first member is always positive and gives the *average systematical increase* of q as the result of stellar perturbations; the second member

[1] This assumption is not exactly fulfilled, because φ depends upon α, β, γ: the vector v_a is at right angles to the vector R.

is symmetrically negative and positive and cancels out in the mean; this member, being proportional to Δv_a, which was assumed to be the projection of a random vector Δu on v_a, may be used to estimate the dispersion of $\Delta q = q_1 - q$ around its average value given by the first member of the formula. Neglecting the dispersion in Δu, we get a minimum spread of Δq, according to the frequency function

$$\sin \varphi \, d\varphi = dt, \tag{16}$$

where

$$t = \frac{\Delta q}{2\Delta q_0} = \cos \varphi$$

and Δq_0 is the absolute value of the second member in (15).

Now, the main spread in Δu, according to (11), depends upon the factor $\sin \gamma$, for given ξ and R, or given $\varepsilon = \frac{\xi}{R}$, except when $\varepsilon = 1$ which is a peculiar case. To allow for this spread, we have to put, instead of t,

$$t_1 = \frac{t}{\sin \gamma} \times \text{constant};$$

the frequency function of γ is $\sin \gamma$. The frequency-function influenced by the spread in Δu will be

$$\int_{\gamma=0}^{\gamma=\pi/2} dt_1 \times \text{frequency-function of } \gamma$$

$$= \int_0^{\pi/2} \frac{dt}{\sin \gamma} \cdot \sin \gamma d\gamma \times \text{constant} = dt \times \text{constant}.$$

Thus we see, that the peculiarity of the spread in Δu is such that it does not much influence the spread in t which is determined by (16). According to this formula, all values of Δq from $-2\Delta q_0$ to $+2\Delta q_0$ are equally probable.

Equation (3′), upon which the validity of our formulae depends, may be represented as

$$\frac{P}{t_0} = 1,11 \frac{\sqrt{R}}{\varepsilon}.$$

For practically important values of ε it may be shown that $\frac{P}{t_0}$ is a great number for R exceeding 1000 a. u. The extreme limit of a rough applicability of our formulae may be estimated at $R > 300$ a. u.

The target area for an approach at a distance $\xi = \varepsilon R$ may be put equal to

$$\sigma = F \cdot \pi\xi^2 = F\pi\varepsilon^2 R^2. \tag{17}$$

The factor F is equal to 2 for $\varepsilon \leq 0.5$ and decreases slowly for greater values of ε toward 1; this factor depends upon the size of the overlapping fraction of the areas of two circles, each of radius ξ, centered at sun and meteor, the distance between the centers being thus R. The formula for F is

$$F = 2 - \frac{2\alpha}{\pi} + \frac{\sin 2\alpha}{\pi},$$

where

$$\cos \alpha = \frac{1}{2\varepsilon}.$$

The probable number of passages closer than $\xi = \varepsilon R$ within the time t years is given in the present case by

$$N_\varepsilon = 3.10^{-18} \cdot F\pi\varepsilon^2 R^2 \cdot 2\pi t = 0,60.10^{-16} F\varepsilon^2 R^2 t. \tag{8'}$$

Table 1 gives the results of a sample computation for $q = 1$.

In this table $\varepsilon = \frac{\xi}{R}$, where ξ is the distance of closest approach either to meteor, or to sun, and R is the aphelion distance; q_1 is the changed perihelion distance; the average increase, $q_1 - q$, does not depend upon q, the initial perihelion distance, unless q is not small as compared with R. The effective limits of q may be somewhat surpassed in the actual case, but for practical purposes it is safe to assume that the maximum and minimum values of q_1, given in the table, symmetrical with respect to "q_1 average," represent the extreme limits of variation of q, and that the frequency function of q_1 is constant with respect to q_1, that is, that equidistant intervals of q_1 are equally probable, to their extreme limits. The negative values of "q_1 minimum" are introduced as a matter of convention, to indicate that the possible values include $q_1 = 0$. Thus, from the table we infer that for $\varepsilon = 0.125$ and less, the changed perihelion distance is always greater than $q = 1$. For $\varepsilon = 0.16$, only the fraction $\frac{1 - 0.9}{36.1 - 0.9}$ or 0.003 will have $|q_1|$ less than 1; for $\varepsilon = 0.25$ the same fraction is $\frac{2.0}{20.8}$ or ten per cent, and the

TABLE 1.

PERTURBATIONS OF THE PERIHELION DISTANCE.

$q = 1$

ε	0.1	0.125	0.16	0.20	0.25	0.32	0.40	0.50	0.64
F_ε^2	0.0200	0.0312	0.0512	0.0800	0.125	0.205	0.320	0.500	0.770
q_1 effective limit max.	73.6	51.4	36.1	25.1	18.2	12.8	9.44	6.97	5.06
q_1 effective limit min.	19.6	8.6	0.9	−1.3	−2.6	−3.0	−2.76	−2.27	−1.66
q_1, average	46.6	30.0	18.5	11.9	7.8	4.92	3.34	2.35	1.70
R, for $N_\epsilon = 1$ and $t = 3.10^9$ years	17000	13500	10500	8300	6700	5200	4100	3300	2700
N_ϵ for $R = 2000$ and $t = 3.10^9$ y.	0.014	0.022	0.036	0.058	0.09	0.15	0.24	0.37	0.55

ε	0.80	1.0	1.28	1.6	2.0	2.56	3.2	4.0	5.12
F_ε^2	1.11	1.61	2.43	3.56	5.28	8.06	12.3	18.8	29.3
q_1 effective limit max.	3.81	2.91	2.22	1.82	1.52	1.33	1.21	1.13	1.09
q_1 effective limit min.	−1.07	−0.53	−0.06	0.26	0.52	0.69	0.79	0.87	0.91
q_1, average	1.37	1.19	1.081	1.038	1.016	1.0065	1.0028	1.0011	1.0005
R, for $N_\epsilon = 1$ and $t = 3.10^9$ years	2240	1860	1510	1250	1020	830	670	540	430
N_ϵ for $R = 2000$ and $t = 3.10^9$ y.	0.80	1.15	1.76	2.56	3.84	5.86	8.9	13.8	21.7

fraction of perturbed orbits which will approach the sun to a distance closer than $|q_1| = 0.25$ a. u. is $\frac{0.5}{20.8} = 2.5$ per cent. From ε somewhat less than 0.20 to about 1.3, a certain fraction of the perturbed objects may hit the surface of the sun, when $|q_1| < 0.0045$; for $\varepsilon = 1.28$ this fraction is $\frac{0.0045 \times 2}{2.28}$ or one in 250. The 6th and 7th lines of the table refer to data connected with N_ε as defined by (8′); the probable fraction of objects which in the indicated interval of time have not undergone an approach, defined by the given value of ε, or closer, is

$$p_\varepsilon = e^{-N_\varepsilon}. \tag{9'}$$

Thus, from the table we infer that at $R = 10500$ the fraction $1/e$ of all objects has not suffered from approaches closer than $\varepsilon = 0.16$ or 1600 a. u. within 3000 million years; or that at $R = 2000$ a. u. the fraction $e^{-2.66}$ or about 8 per cent has not suffered changes from approaches closer than $\varepsilon = 1.6$ or 3200 a. u. The table, though constructed for $q = 1$, may be easily adapted to any other value of q, when q remains small compared with R; in agreement with formula (15), we must only add $q - 1$ to "q_1 average," and change the range of the effective limits of q_1 in the ratio $\sqrt{q}$. All the other data of the table remain unchanged. Any changes in N_ε, R or t in the last two lines of the table are to be made in agreement with formula (8′).

The most remarkable fact revealed by Table I is the tendency of the perihelion distance to increase continually. After a sufficient interval of time the perihelia will shift out of the sphere with radius $q = 1$, and even out of the known planetary limits of the solar system. The systematic increase in q, depending upon the probability of an approach, increases with the aphelion distance. Thus, for R sufficiently great, we may expect that most of the meteors, or comets, have already gone out of the region of observability. This limit of R may be estimated from the table at 10000 to 20000 a.u., assuming an age of 3.10^9 years, and would correspond to periods of revolution from 400,000 to one million years. It is not very probable that many of the observable meteors have periods of revolution greater than these figures, unless the total number of such meteors in the solar system is very great. In the latter case, because the spread in q_1 increases as $\sqrt{q}$, a small fraction of meteors possessing a large q may be changed by perturbations into those with a small value of q_1 and such objects may thus enter observable regions of the solar system. A state of

statistical equilibrium will be installed when the number with small q changed into big q_1 will be balanced by the number with big q changed into small q_1. Because of the said systematical trend of the perturbations in q, the equilibrium will take place when the number with great q will be greater than the number with small q. It is easy to see that this means also a certain increase of concentration of meteoric matter in the nonobservable regions of the solar system, although the concentration is only a relative one, being counter-balanced by dilution in a greater volume of space. If, however, among our observable objects there is a sensible proportion of meteors with R exceeding 10000 a. u., it may be only the result of a very peculiar distribution of meteoric matter at the outskirts of the solar system—a kind of a meteoric cloud, or shell, of a density much higher than may be anticipated from the observed frequency of solar meteors caught on the surface of the terrestrial atmosphere. Unless this is the case, we may expect that the majority of solar meteors we observe comes from distances less than 10000 a. u.

This supposition means that an average solar meteor must have completed at least many thousand revolutions during its life time, being thus exposed for a total interval of a thousand years or more to comparatively high temperatures during perihelion passages. This deduction may have some bearing on the determination of the age of meteorites from their helium content, and upon the gradual loss of the more volatile constituents of the meteors. The heating effect of the sun is increased through the character both of the systematic and accidental variation of q. A meteor met by the earth must have q less than 1, and because of the systematic increase of q with time the probability is that in the past history of the meteor the average q was still smaller. Furthermore the variation of q is such that small changes of q occur much more often, and are more symmetrically distributed around the original value, than big perturbations. The result is a continuous unsteadiness of the orbit, a certain property of "mixing," so that the probability is considerable that the meteor, before being caught by the earth, or before "jumping out" as the result of a big perturbation, will undergo some very close approaches to the sun.

The stability of the orbit increases as the aphelion distance decreases. At $R = 2000$ a. u., for a period of revolution of 30000 years, the perturbations during $t = 3.10^9$ years are sensible, but not so revolutionary. Below 2000 a. u., the majority of the orbits may be regarded as statistically stable during the adopted interval of time.

The perturbations of the inclination have much in common with the perturbations in perihelion distance except for the systematical character. When the perturbations are small, the *average change* of inclination is given by $\overline{tg\Delta i} = \frac{q_1 - q}{2q}$; for stronger perturbations the conditions are more complicated. In the following table is given the average expected change in inclination for $t = 3.10^9$ years.

R	100	250	500	1000	1500	2000
$\overline{\Delta i}$	$\pm 5.°5$	$\pm 14°$	$\pm 28°$	$\pm 55°$	$\pm 82°$	$\pm 110°$

For other intervals of time $\overline{\Delta i}$ is proportional to $\sqrt{t}$. An average change $\overline{\Delta i} = \pm 90°$ already means a close approach to uniform distribution. According to statistics by Jantzen, 395 comets (1909) showed a more or less random distribution of the inclinations, with a slight preponderance of direct motions over retrograde ones; with an age of from 10^9 to 3.10^9 years, this would correspond to an average aphelion distance of 1500–2000 a. u., or a period of revolution of 20000–30000 years. For greater aphelion distances the distribution of inclinations should be practically uniform, being smoothed out by perturbations. Meteors seem to show a much stronger preference for direct motion,[1] which could be preserved in the course of time only at a smaller effective aphelion distance, about 1000 a. u., corresponding to a period of about 10000 years.

Conclusions.

1. The possible limits of the solar system, as set by stellar perturbations during the time elapsed since the origin of the system, may be estimated at 10^6 astronomical units; whether the actual limits reach so far, depends upon the presence of bodies at these distances.

2. Meteors, or comets of an aphelion distance exceeding 10000 astronomical units, are not very likely to occur among the observable objects, because of the rapid increase of the average perihelion distance due to stellar perturbations.

3. From the scarce data relating to the observed nonuniformity of the orbital inclinations, the effective aphelion distance of observable comets is estimated at 1500–2000 astronomical units, and of observable sporadic meteors at 1000 astronomical units.

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[1] E. Öpik, H. C. 355, p. 9, 1930.

4. At the aphelion distance of 1000 a. u., the orbit of a meteor during 3.10^9 years is comparatively stable; the average heating effect of the sun on an observable meteor corresponds in this case to from 100000 to 300000 approaches at the actual perihelion distance, or to about 30000 years of continuous heating to a temperature exceeding 0°C in vacuo, which may be sufficient for a stony meteorite to lose all its helium content. It is therefore probable that stony meteorites of solar origin yield very low ages according to the helium-radium method.

5. As the result of stellar perturbations, the number of meteors and comets of a given constant aphelion distance of 2000 a. u. or greater must increase with increasing perihelion distance, to a certain limit; the rate of relative increase in the number must be greater for greater aphelion distances. For objects with an aphelion distance below 1000 a. u., stellar perturbations could not change appreciably the original distribution of the orbital elements within 3.10^9 years.

The present paper gives only an approximate sketch of the situation. The approximate results are, however, quite sufficient to serve as bases for certain numerical estimates which appear to be useful and even necessary in the course of the systematic meteor research undertaken at Harvard Observatory under the initiative and organization of Dr. Shapley.

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